Algebra for Gifted Visual-Spatial Learners

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In the early 1980s, Dr. Linda Silverman discovered an over-arching division of learning characteristics into two categories, which she termed "auditory-sequential" and "visual-spatial." Her findings were based on extensive research.

> "...(W)e have amassed data on...learning modes, behavior patterns, and personality characteristics that appear to be correlated with high visual-spatial abilities. We have found clusters of traits appearing with such regularity that we have come to believe that they are directly related to a visual-spatial orientation to learning." (Silverman, 1982/1989, p. 15)

While each student shows a natural preference or inclination towards either the visual-spatial or the auditory-sequential learning style, it appears that they are not mutually exclusive. Hence, some gifted learners may be predominantly visual-spatial but show strong auditory-sequential abilities. A comprehensive discussion of the differences in these two learning styles is beyond the scope of this paper. But for a fuller treatment, the reader should consult Dr. Linda Silverman's new book, *Upside-Down Brilliance: The Visual-Spatial Learner*.

In the normal classroom setting, whenever standard teaching methods are used, the visual-spatial learner faces a number of disadvantages. School is predominantly an auditory environment in which the curriculum, textbooks, classroom management techniques, teaching methods, and the teachers themselves are sequential. The over-reliance on auditory-sequential methods in secondary mathematics works against the visual-spatial learner. Typically, there is little use made of manipulatives, in contrast to mathematics instruction at the elementary level. Moreover, there is infrequent use of demonstrations and participatory activities to get students up out of their desks and infrequent use of graphics technology, such as graphing calculators or appropriate computer software.

It is seldom appreciated, however, that gifted visual-spatial students possess certain natural advantages in learning mathematics from algebra to calculus and beyond. They see whole concepts quickly, find patterns easily, think graphically, and understand dimensionality. In fact, certain concepts of advanced mathematical reasoning may be only genuinely accessible through the visual-spatial channel.

Unfortunately, many of these visual-spatial students may have switched to compensatory learning strategies in arithmetic classes in the elementary grades that rely on auditory and sequential skills, even though auditory-sequential processing is not their strength. An example of such a switch would be memorizing math facts and formulas as if they were recipes, without delving into their real meaning. At a more advanced level of mathematics, those strategies may no longer be appropriate and may even be a hindrance.

The discovery that tapping into visual-spatial strengths is necessary for an understanding of advanced math concepts may come as a shock to these students. For them, an awakening to the visual-spatial aspects of algebra often leads to rapid progress in advanced algebra, trigonometry, and calculus, and also in related fields like physics.

In order to succeed in the verbal and sequential world of the secondary mathematics classroom, students with such a relative deficit have a tendency to rely on their considerable auditory-sequential skills, instead of leading from their strengths. They develop compensatory strategies that de-emphasize their visual-spatial abilities. Through disuse, their visual-spatial skills might even begin to atrophy. In some remarkable instances, the transformation becomes so acute that the student is not even aware of his or her preferred learning style. By careful mentoring work with these students, it is possible to put them back in touch with themselves and their innate learning style. An analogy would be left-handed golfers who were told for years that golf is a right-handed game. If they are able to make the switch, they might still develop some proficiency with the game, but they will probably not achieve the level they could attain if they had practiced as left-handers.

Those unfortunate gifted visual-spatial learners who lack the ability to compensate, because they lack the necessary auditory-sequential skills, confront an entirely different set of problems. They can be said to have a real, not a relative, auditory-sequential deficit. As a result, in most instances, their giftedness is probably not identified properly by any of the instruments or methods currently in use. If they hope to succeed in the classroom, they must find ways to translate the material presented into a format that can be assimilated. Anyone who has taken a class in a foreign language without being completely fluent has some inkling of the difficulty of the task and the energy that must be expended in actively and simultaneously translating back into the mother tongue. To the extent that these visual-spatial students do not adapt, their performance in the classroom setting suffers.

In mathematics, we face a special challenge when it comes to the gifted visual-spatial learner. Much of higher mathematics is only truly accessible through the visual-spatial channel. However, even before these students get to such a level, we may lose many of the ones with real auditory-sequential deficits. Those with only a relative deficit are retrained while they are learning more elementary mathematics to ignore the very skills and abilities that they will need for brilliance later. In order to handle that challenge, educators must develop successful classroom teaching techniques that put those with a *relative* auditory-sequential deficit back in touch with their relative strengths and help those with a *real* auditory-sequential deficit to find ways to survive in the mathematics classroom.

Visual-Spatial Strengths

It is possible to elaborate a set of four major descriptors for those characteristics of visual-spatial learning that are exhibited with subject material from algebra to calculus (see also Arnheim, 1969; Dixon, 1983; Silverman, 2002; West, 1991). Those descriptors also track closely with the research work underpinning the development of the *Visual-Spatial Identifier* (Haas, 2001; Silverman, 2002) and with the author's eleven years experience teaching, tutoring, and mentoring gifted students in secondary mathematics. We can detect major differences in how these students use imaging, approach conceptualization of the subject matter, employ strategies in solving problems, and find and use patterns.

Imaging. Visual-spatial students learn better by seeing than by listening. Even when listening to an oral presentation, they are likely to be actively creating visual images in order to input and process the information being presented. For them such activities as gazing at the ceiling or out the window or doodling in their notebooks can actually assist in their learning.

These students have the ability to grasp concepts in multiple dimensions, typically three but often more. They easily understand changes in perspective in problems, such as movement, translation, reflection, or rotation. They have equal facility in shifting the concept being studied or the point from which it is viewed. Such ability provides a direct channel through which they can access advanced mathematical concepts and their physical applications. In general visual-spatial learners lack a more normal sense of time as an ordered sequence; however, they are able to view math functions as movement through space and time. In this way, they are able to understand the concept of parametric equations at a relatively early stage in their secondary math education.

While much of algebra is usually presented as a sequential step-by-step process, in most instances there are direct graphical representations available to explain the same material. Technology is currently available that can facilitate the student's exploration of those graphical aspects. For example, software for the PC can perform real-time rotations of three-dimensional objects and provide matrix representations of phenomena in more than three dimensions. Hand-held technology, such as the graphing calculator, which can plot functions automatically, is only about four to five years behind what is generally available on PCs. For the visual-spatial student, an explosion of math interest and insight usually occurs upon first contact with a graphing calculator.

Conceptualizing. Visual-spatial students are holistic learners who grasp whole concepts rather than individual facts. They synthesize and construct conceptual frameworks to show connections between a particular topic and the rest of the subject. They often experience difficulty in memorizing formulas or math facts or in learning skills when these skills are taught as an unconnected series of isolated elements.

Teachers often use mastery of arithmetic and computational skills in elementary math as a gatekeeper to more advanced material. But this teaching strategy often works against visual-spatial learners. Typically, they are not very attentive to detail. They are prone to computational errors or missing a negative sign. For the gifted visual-spatial learner, rote repetition and memorization can lead to boredom, which may be reflected in declining grades, even on simple material.

Because of their whole-concept approach to learning, they are not distracted by detail but enjoy finding out how to incorporate it into a larger framework. Rather than shun complexity, they thrive on it. They can handle simultaneity, ambiguity, and complex interactions within a subject. Visual-spatial students also see connections between mathematics and other disciplines. Those connections include the more mundane links with science, but can easily extend to music, dance, and art.

Problem-Solving. Visual-spatial learners are divergent thinkers, who prefer unusual solution paths and multiple strategies for problem-solving. They enjoy playing around with a problem and sometimes finding five or more solution strategies. The process of getting there is more important to them than any answer.

Because they see the whole concept first, they are able to reason from conclusions backwards, often skipping intermediary steps. It should not, however, be inferred that this back-to-front reasoning is itself some variant of sequential processing. On the contrary, these students are particularly adept at solving mazes by starting in the middle or seemingly anywhere at all.

In the absence of a step-by-step sequential approach, it may appear that visual-spatial students have leaped to conclusions "intuitively." Some teachers insist that only students who can show their work really understand the concepts. VSLs are often not able to articulate to others how they arrived at a conclusion, even if the process seems clear to them. This can lead to doubt in the student's mind and in the teacher's mind. Sometimes, the student may fear not being able to replicate a problem-solving process. And worse yet, the teacher may suspect the student either of guessing or cheating, especially if the student fails to show work on the problem.

Pattern-Seeking. Not only do visual-spatial students excel at finding patterns in numbers but they also at times seem driven to finding those patterns in order to make sense of the mathematical principles they embody.

They are also adept at functional reasoning, in which two or more sequences of numbers are related through a mathematical operator called a function. They are good at finding patterns and functional relationships in numbers and investigating them. The importance of this facility with functions cannot be over-emphasized since a thorough understanding of the theory of functions is the key that unlocks the door to advanced mathematics.

As mentioned previously, visual-spatial learners easily understand and portray mathematics as graphical representations. They are able to see functions graphically, without even plotting individual points. However, visual-spatial learners do not necessarily excel in graphing, especially if the subject is presented as a series of ordered pairs of "x" and "y" values in a "T" chart.

Potential School Problems

From the foregoing, it should be clear that while visual-spatial students have a number of distinct advantages in learning secondary mathematics, they face formidable obstacles in the typical high school classroom. Classrooms are largely verbal environments. Instructions are given verbally; questions and answers are verbal, sometimes with follow-up on the blackboard; and drill, practice, and challenge exercises are usually presented verbally.

To survive in the secondary mathematics classroom, students generally must have strong sequential processing skills. For example, algebra is usually taught as algorithms, which are solved by step-by-step operations. In solving equations, students are required to show each step and to check by plugging answers back into the original equation. Students must also be able to reproduce and explain their problem-solving process. In learning to solve equations, there is an over-reliance on rote drill and practice. Formulas and equations are usually memorized.

Because of geometry's reliance on sketches and drawings of visual objects, it might be assumed that geometry does not suffer from the same static, auditory-sequential approach as algebra. Indeed, many students report an entirely different personal experience with geometry, compared to algebra, and that difference might be attributable to geometry's more visual-spatial basis. However, much of geometry is taught as a logical and deductive construct, with definitions leading to postulates and on to theorems and proofs. Traditional two-column proofs, in which each assertion in the left column must be matched with a justification in the right column, are highly-sequential and, therefore, a particular problem for visual-spatial learners.

A Sampler of Algebra Topics

The rest of this article examines a sampler of algebra topics, listed below, with ideas for presenting them in ways designed to make sense to the gifted visual-spatial learner.

Absolute value and linear shifts
Functions, music, and dance
Mazes and reverse logic
Velocity, acceleration, and distance
Parametric equations
Trigonometry without triangles

Absolute Values and Linear Shifts

When students are asked to solve inequality problems like $|x - 3| < 2$ using traditional algebraic methods, they learn to divide the problem into two solution streams, solve, and then recombine them as a single answer, in this case namely, $1 < x < 5$. For the visual-spatial learner, however, recognition of the pattern is the key to understanding the concept. They can easily visualize the graph on a number line of the solution to $|x| < 2$. Then, it is a rather simple matter to take the center of that plot, at $x = 0$, and put it in motion, letting it slide to the right until it comes to rest at $x = 3$.

Functions, Music, and Dance

As previously mentioned, functions are operators or combinations of operators that relate one set of numbers (the inputs) to another set of numbers (the outputs). The concept can be illustrated with a drawing of a function box. An example of a set of inputs and outputs could be $\{1,2,3,4\}$ and $\{3,5,7,9\}$; in this case, the function is $2x + 1$.

At this point in the instruction, the teacher can announce that the next class period will be a dance party, in which students should bring their own music. The only catch is that before they can put on their music, every one of them must first try out the teacher's two dances, a Viennese waltz and 1950's rock 'n roll.

The teacher can come back to the experience later during the year as a touchstone for work on functions. In essence, dancing with partners demonstrates how pairs move together in ways consistent with, and under the constraints of, the music. In this case, the music is the function. Dance utilizes the natural affinity that many kinesthetic and spatial learners feel for music and movement through space. Dancing can serve to activate the students' math interest. Using music in this way depicts math as an art form and encourages students to look beyond static pencil and paper representations.

Mazes and Reverse Logic

The idea of using mazes to demonstrate reverse logic is quite straightforward. First, all the students create their own mazes to share with the rest of the class. With duplicated sets of everyone's mazes, they have races to see who can solve them the quickest. Consistent winners are then asked to share their secret, which usually is that they rarely go from start to finish. Amazingly, some winners will even report starting somewhere in the middle.

Many topics in mathematics require an ability to reason and work in reverse. For example, the algorithm for solving a linear equation for the independent variable uses reverse logic and is, in fact, called the inverse function.

$$2x + 1 = y \qquad\qquad \frac{x - 1}{2} = y$$

In another example, the process of factoring just reverses the multiplication of one binomial by another. Those who can visualize where a polynomial came from find factoring easy. In a more advanced example, differential and integral calculus are also in some sense the reverse of each other, and are formally linked to one another through the Fundamental Theorem of Calculus.

Velocity, Acceleration, and Distance

The basis for this section is the motorboat problem. Students are provided with a graph showing how the motorboat's velocity (in meters/second) changes over elapsed time (in seconds). They are asked to identify places on the graph where the velocity is constant and also find when it is increasing and decreasing.

Distance has a more curious relationship to the graph. When velocity is constant and positive, the students can figure out how far the boat travels during that time period, by using the formula $d = (r)(t)$. But, coincidentally, the graphic representation of distance is the area of a rectangle under the line, with a length of "t" and a height of "r."

When velocity is increasing uniformly, students can find the distance travelled by averaging velocity and finding the area of the trapezoid under the line (or triangle if the starting velocity was zero). The students can then be split into teams to try to calculate the total distance the boat travels cumulatively over the entire time depicted on the graph. In this exercise, the visual-spatial students will be able to demonstrate some advantages to their learning style in interpreting the graph.

Based on the same principles as the classroom motorboat problem, the students break into teams of three and get one of the parents to volunteer to drive. They can then replicate the motor boat graph with a real experiment, using a car, its speedometer, and a watch. They can check their results with the car's odometer.

Parametric Equations

In beginning algebra, we normally speak of "x" as an independent variable and "y" as a dependent variable. However, values for "x" and "y" can both be independent of each other but each dependent on a third variable, like time. To demonstrate the principle, the teacher can help students explore the projectile motion of a ball thrown horizontally off a cliff, with two components to its motion.

By using time as the driving variable, we can put otherwise static functions into motion. If a big floor the size of a gym is available with linoleum tiles, it is possible to organize a class project on parametric motion. The linoleum tiles provide a convenient grid for the coordinate plane. Students moving along each axis simulate each variable as a function of time. A third student must travel across the floor so as to stay perpendicular to the two students on the axes. The speed of his motion along his path describes the parametric function.

Trigonometry Without Triangles

Most treatments of trigonometry start with the definitions of the six ratios in a triangle. Instead, with a background in parametric functions, students can begin by observing a simple pendulum, like a grandfather clock. A ball on a string in a circular stairwell works even better. By grouping students on each axis and asking them to describe and graph the distance of the ball from the center, as a function of time, students can actually see the sine wave and cosine wave functions. The teacher can then set the ball into a circular motion so that, by shifting their line of sight, the students can switch from one function to the other and visualize both simultaneously, as parametric functions.

Conclusion

These are just a few of the many examples possible for alternative methods of instruction. In a similar way, a whole course outline of algebra topics can be developed that rely principally upon a visual-spatial presentation. If the teacher prefers or feels more comfortable with the more traditional presentational methods, it is still possible, and even desirable, to use visual-spatial methods as a supplement to ensure that students without natural auditory-sequential strengths are not disadvantaged in learning the material.

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